64-71 $\sum_{n=0}^{\infty}(x-1)^n = 1 + (x-1) + (x-1)^2 \ldots = \frac{1}{1-(x-1)}$ $= \frac{1}{2-x}$

69. $|r|<1$ $r = x-1$ $|x-1|<1$

A

$-1 < x-1 < 1$

70 E $0 < x < 2$

71. $\int \frac{1}{2-x}\,dx = -\ln|2-x| + C$

$-\ln \frac{|x-2|}{2} = -\left(\ln|x-2| - \ln 2\right)$

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63. $\int \frac{1}{x} = \int \frac{1}{1+(x-1)} = \int 1 - (x-1) + (x-1)^2 - (x-1)^3 \ldots$

$a=1$ $r=-(x-1)$

$\ln x = x - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} \ldots$

48. 

$a=8$ $r=.6$

$\frac{8}{1-.6} - 4$

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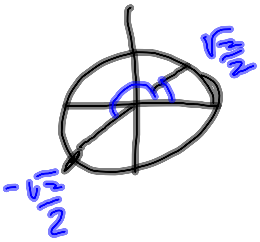
17. $$\sum_{n=0}^{\infty} \sin^n\left(\frac{\pi}{4}+n\pi\right) = 1 + \sin\left(\frac{5\pi}{4}\right) + \sin^2\left(\frac{9\pi}{4}\right)$$



$$= \underset{a}{\underset{\uparrow}{1}} \underset{r}{\underset{\uparrow}{-\frac{\sqrt{2}}{2}}} + \dots$$

$$= \frac{1}{1+\frac{\sqrt{2}}{2}} \quad \frac{2}{2} = \frac{2}{(1+\sqrt{2})} \frac{(1-\sqrt{2})}{(1-\sqrt{2})}$$

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21. $$\sum_{n=0}^{\infty} 2^n x^n = 1 + 2x + 4x^2 \dots$$

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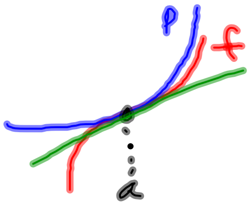
9.2 Taylor Series

given $f(x)$, find its power series



$p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 \dots$

center of interval of convergence $= a$

$f(a) = p(a)$ let $a = 0$ Maclaurin Series

$f(0) = p(0)$ so $a_0 = f(0)$

$f'(0) = p'(0)$ $p'(x) = a_1 + 2a_2 x + 3a_3 x^2 \dots$

$f'(0) = a_1$

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$f''(0) = p''(0)$ $p''(x) = 2a_2 + 6a_3 x \dots$

$f''(0) = 2a_2$

$\frac{f''(0)}{2} = a_2$

$p'''(x) = 6a_3 + \dots$

$f'''(0) = p'''(0)$

$6 = 3 \cdot 2 \cdot 1$ $\frac{f^3(0)}{6} = \cancel{6}a_3$

$a_4 = \frac{f^4(0)}{4 \cdot 3 \cdot 2 \cdot 1}$

coefficients $\rightarrow$ $\boxed{a_n = \frac{f^n(0)}{n!}}$ n^{th} derivative

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$$f(x) \approx f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f^3(0)}{3!}x^3 \ldots$$

$$\approx \sum_{n=0}^{\infty} \frac{f^n(0)}{n!}x^n$$

e^x - find the Maclaurin series

$f(x) = e^x$ $f(0) = 1$
$f'(x) = e^x$ $f'(0) = 1$
$f''(x) = e^x$ $f''(0) = 1$
$f^3(x) = e^x$ $\vdots$
$f^4(x) = e^x$ $\vdots$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} \ldots$$

$$e^x = 1 + x + \frac{3x^2}{3!} + \frac{4x^3}{4!}$$

$$= 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} \ldots$$

$4 \cdot 3 \cdot 2 \cdot 1$

let $x = 1$ $\quad e = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2 = 2!} + \frac{1}{6 = 3!} + \frac{1}{4!} + \frac{1}{5!} + \ldots$

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$$e = \sum_{n=0}^{\infty} \frac{1}{n!}$$

$f(x) = \sin x$ $\quad f(0) = 0$
$f'(x) = \cos x$ $\quad f'(0) = 1$
$f''(x) = -\sin x$ $\quad f''(0) = 0$
$f^3(x) = -\cos x$ $\quad f^3(0) = -1$
$f^4(x) = \sin x$ $\quad 0$
$f^5(x) = \cos x$ $\quad f^5(0) = 1$
$f^6(x) = -\sin x$ $\quad 0$
$f^7(x) = -\cos x$ $\quad -1$
$\vdots$

$$x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \ldots$$

$P_7(x)$

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